Pergamon

J. Aerosol Sci., Vol. 26. Suppl 1, pp. S289–S290, 1995
Elsevier Science Ltd
Printed in Great Britain
0021–8502/95 $9.50 + 0.00

Geometrical optics approximation for coefficients of expansion of phase function in Legendre polynomials

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KEYWORDS

Radiative transfer theory, Phase function, Aerosols

The expanding of phase functions $x(\theta)$ of aerosol media in Legendre polynomials $P_l(\theta)$

$$x(\theta) = \sum_{l=0}^{N} x_l P_l(\theta) \tag{1}$$

is frequently used to solve the radiative transfer equation and to find radiative characteristics of aerosol layers.

The number of terms in eq. (1) is about 2ρ, where $\rho = ka$, $k = 2\pi/\lambda$, a is the radius of particle, λ is the wavelength. the coefficients x_l can be obtained from the following formula:

$$x_l = \frac{2l+1}{2} \int_0^{\pi} x(\theta) P_l(\theta) \sin\theta d\theta \tag{2}$$

There are some difficulties to use Eq. (2) for large ($a \gg \lambda$) particles, when one should calculate function $x(\theta)$ with the Mie theory. This problem can be simplified by using geometrical optics approximation for $x(\theta)$. The task of this paper is to derive simple formulae for calculation of the values of x_l within the framework of the geometrical optics approximation.

After substitution the geometrical optics approximation for the function $x(\theta)$ in Eq. (2) the following formula can be obtained:

$$x_l = \frac{\sigma^d_{sca} x^d_l + \sigma^G_{sca} x^G_l}{\sigma^d_{sca} + \sigma^G_{sca}} \tag{3}$$

where

$$\sigma^d_{sca} = \pi a^2, \qquad \sigma^G_{sca} = \omega \pi a^2 \tag{4}$$

$$x^d_l = (2l+1)F(t), \quad x^G_l = (2l+1)\sum_{s=0}^{l} B_s \varphi_s(\alpha) \tag{5}$$

Here

$$\omega = \frac{1}{2}\sum_{j=1}^{2}\int_0^{\pi/2}\left(R_j + \frac{(1-R_j)^2 e^{-C\xi}}{1-R_j e^{-C\xi}}\right)\sin 2\tau d\tau \tag{6}$$

$$F(t) = \frac{2}{\pi}(\arccos t - t\sqrt{1-t^2})U_+(t), \quad B_s = \frac{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(l-s+\frac{1}{2}\right)}{\pi\Gamma(s+1)\Gamma(l-s+1)} \tag{7}$$

$$\varphi_s(\alpha) = \int_0^{\pi/2}\frac{\Phi_{1s}(\alpha,\tau)+\Phi_{2s}(\alpha,\tau)}{2\omega}\sin 2\tau d\tau \tag{8}$$

$$\Phi_{js}(\alpha,\tau) = \frac{D_j(\alpha,\tau) + 2R_j^2 e^{-C\xi}(e^{-C\xi} - \cos 2\alpha\tau')\cos 2\alpha\tau}{1 - 2R_j e^{-C\xi}\cos(2\alpha\tau') + R_j^2 e^{-2C\xi}} \tag{9}$$

$$D_j(\alpha,\tau) = R_j(1-e^{-2C\xi})\cos 2\alpha\tau + (1-R_j)^2 e^{-C\xi}\cos 2\alpha(\tau-\tau') \tag{10}$$

Γ is the gamma function, $U_+(t) = 0$ at $t > 1$ and $U_+(t) = 1$ at $t \leq 1$, $t = l/2\rho, c = 4\gamma\rho$, $\cos\tau' = \cos\tau/n, m = n - \mathrm{i}\gamma$ is the complex refractive index of particles, $\xi = \sqrt{1-\cos^2\tau/n^2}, R_j$ is the Fresnel reflection coefficients, $\alpha = l - 2s$.

Eq. (3) can be used to calculate the coefficients x_l for large ($\alpha \gg \lambda$) single particles. It was generalized on spherical polydispersions also. Note, that coefficients x_l^G decreases rapidly at $l \to \infty$ and $x_l^G \ll x_l^d$ as $l > 10$ for aerosol media. So at large l we have (see Eq. (3)) $x_l = x_l^d/(1+\omega)$. The maximum of the function $x(l)$ is located at $l_m = \pi\rho/4$ and $x(l_m) = l_m/(1+\omega)$.